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Unitary Operator and Zero-Point Fluctuation Properties of a Polariton System

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In a model polariton system, we find a unitary operator which transforms canonically from the uncoupled states to the coupled states of the phonon–photon system. We investigate the ground-state properties of the system and show that when the polariton system is in its lowest energy state (the vacuum state), which means that no radiation occurs, the phonon and photon subsystems can exhibit nonclassical behavior.

The study of the behavior of coupled phonon–photon systems has long been an active research area in condensed matter physics because of its significance in determining the phonon properties. Recently, efforts have been made to study the nonclassical nature of phonons using the ideas of quantum optics. For example, squeezed phonon states have been studied theoretically (Hu and Nori, 1996, 1997) and experimentally (Garret *et al.*, 1997). They are also interesting in connection with a model solid-state system or a model polariton system (Ghoshal and Chatterjee, 1995, 1996). A polariton is a phonon–photon complex which can be formed when light falls on a solid material and interacts with the vibrating lattice. For the model polariton system, Ghoshal and Chatterjee (1996) have given a transformation relation between the phonon–photon operators and that of the polariton operators to diagonalize the Hamiltonian of the system. In this paper, we show that the Ghoshal–Chatterjee transformation can be represented as an explicitly unitary operator form. Thus, a relation between the coupled and uncoupled states of the phonon–photon system is found. We demonstrate that a new ground state of the polariton system proposed by Wang *et al.* (1997) is just the polariton

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vacuum state. We also investigate the zero-point fluctuation properties of the polariton system.

We first briefly describe the Ghoshal–Chatterjee model (1996) and its main results. The polariton system involve one mode of the photon field interacting with a single optical phonon mode. The polariton model can be described by the Hamiltonian

$$H = \omega_a a^\dagger a + \omega_b b^\dagger b + k(a^\dagger b^\dagger + ab + a^\dagger b + ab^\dagger) \tag{1}$$

where $a(a^\dagger)$ is the annihilation (creation) operator for the phonon field with frequency ω_a, $b(b^\dagger)$ is the annihilation (creation) operator of the photon field with frequency ω_a, and k is the phonon–photon coupling constant. Using the transformations

$$a = A_1\alpha + A_2\alpha^\dagger + B_1\beta + B_2\beta^\dagger \tag{2}$$

$$b = B_3\alpha + B_4\alpha^\dagger + A_3\beta + A_4\beta^\dagger \tag{3}$$

and choosing suitable parameters

$$A_1 = \frac{A}{2}\left(\lambda_1 + \frac{1}{\lambda_1}\right), \qquad A_2 = \frac{A}{2}\left(\lambda_1 - \frac{1}{\lambda_1}\right) \tag{4}$$

$$A_3 = -\frac{A}{2}\left(\lambda_2 + \frac{1}{\lambda_2}\right), \qquad A_4 = -\frac{A}{2}\left(\lambda_2 - \frac{1}{\lambda_2}\right) \tag{5}$$

$$B_1 = \frac{B}{2}\left(f\lambda_2 + \frac{1}{f\lambda_2}\right), \qquad B_2 = \frac{B}{2}\left(f\lambda_3 - \frac{1}{f\lambda_2}\right) \tag{6}$$

$$B_3 = \frac{B}{2}\left(\frac{\lambda_1}{f} + \frac{f}{\lambda_1}\right), \qquad B_4 = \frac{B}{2}\left(\frac{\lambda_1}{f} - \frac{f}{\lambda_1}\right) \tag{7}$$

$$A = \left(\frac{\sqrt{1+g^2}+g}{2\sqrt{1+g^2}}\right)^{1/2}, \qquad B = \left(\frac{\sqrt{1+g^2}-g}{2\sqrt{1+g^2}}\right)^{1/2} \tag{8}$$

$$\lambda_1 = f\left(\frac{A^2 - B^2}{A^2 f^4 - B^2}\right)^{1/4}, \qquad \lambda_2 = \left(\frac{A^2 - B^2}{A^2 - B^2 f^4}\right)^{1/4} \tag{9}$$

$$f = \sqrt{\frac{\omega_a}{\omega_b}}, \qquad g = \frac{(\omega_a^2 - \omega_b^2)}{4k\sqrt{\omega_a\omega_b}} \tag{10}$$

we can diagonalize the Hamiltonian (1) as

$$H = E_\alpha \alpha^\dagger \alpha + E_\beta \beta^\dagger \beta + E_0 \tag{11}$$

where α, β are new Bose operators in the polariton system. E_0 is the ground-state energy of the system and is given by

$$E_0 = \frac{1}{2}(E_a + E_b - \omega_a - \omega_b) \tag{12}$$

$$E_a = [(\omega_a^2 A^2 + \omega_b^2 B^2) + 4k\sqrt{\omega_a\omega_b AB}]^{1/2} \tag{13}$$

$$E_b = [(\omega_b^2 A^2 + \omega_a^2 B^2) - 4k\sqrt{\omega_a\omega_b AB}]^{1/2} \tag{14}$$

The transformation (2), (3) is canonical since it leaves the commutator invariant, $[\alpha, \alpha^\dagger] = 1$, $[\beta, \beta^\dagger] = 1$, $[\alpha, \beta] = 0$. A theorem of von Neumann (1931) asserts that every canonical transformation can be represented as a unitary transformation. Thus, we can perform

$$\alpha = UaU^{-1}, \qquad \beta = UbU^{-1} \tag{15}$$

where U is a unitary operator and leads to the transformation properties of (2), (3). Our purpose is to look for the form of the unitary operator so as to build a relation between the uncoupled state and the coupled one. The Hamiltonian (11) can be rewritten as

$$H = U H_0 U^{-1} \tag{16}$$

where

$$H_0 = E_\alpha a^\dagger a + E_\beta b^\dagger b + E_0 \tag{17}$$

and the operators α, β and a, b satisfy the following eigenvalue equations:

$$\begin{aligned} \alpha^\dagger\alpha|n_1\rangle_\alpha &= n_1|n_1\rangle_\alpha, \qquad \beta^\dagger\beta|n_2\rangle_\beta = n_2|n_2\rangle_\beta \\ a^\dagger a|n_1\rangle_a &= n_1|n_1\rangle_a\,, \qquad b^\dagger b|n_2\rangle_b = n_2|n_2\rangle_b \\ n_1, n_2 &= 0, 1, 2, \ldots \end{aligned} \tag{18}$$

The diagonalized Hamiltonian (11) means that one has "dressed" the phonons, and the indices α and β specify the two branches of energy spectrum. Let the two-mode states $|n_1 n_2\rangle_{\alpha\beta}$ denote the eigenstates of H and $|n_1 n_2\rangle_{ab}$ the eigenstates of H_0. It is easy to see that $U|n_1 n_2\rangle_{ab}$ are the eigenstates of H. Thus, we find the following transformation:

$$|n_1 n_2\rangle_{\alpha\beta} = U|n_1 n_2\rangle_{ab} \tag{19}$$

Once the unitary operator is obtained, the connection between the uncoupled state and the coupled one of the system can be known.

We now derive the explicit form of the unitary operator. Let us introduce the coordinate operators in the form

$$Q_a = \frac{1}{\sqrt{2\omega_a}}(a + a^\dagger), \qquad Q_b = \frac{1}{\sqrt{2\omega_b}}(b + b^\dagger) \tag{20}$$

The corresponding coordinate eigenstates are given by

$$\left|q_a\right\rangle = \left(\frac{\omega_a}{\pi}\right)^{1/4} \exp\left(-\frac{\omega_a}{2} q_a^2 + \sqrt{2\omega_b} q_a a^\dagger - \frac{1}{2} a^{\dagger 2}\right)\left|0\right\rangle_a \tag{21}$$

$$\left|q_b\right\rangle = \left(\frac{\omega_1}{\pi}\right)^{1/4} \exp\left(-\frac{\omega_b}{2} q_b^2 + \sqrt{2\omega_b} q_b b^\dagger - \frac{1}{2} b^{\dagger 2}\right)\left|0\right\rangle_b \tag{22}$$

From (2)–(7) and (15), we have

$$a + a^\dagger = U[A\lambda_1(a + a^\dagger) + Bf\lambda_2(b + b^\dagger)]U^{-1} \tag{23}$$

$$b + b^\dagger = U\left[B\frac{\lambda_2}{f}(a + a^\dagger) - A\lambda_2(b + b^\dagger)\right]U^{-1} \tag{24}$$

Thus we obtain

$$\begin{pmatrix} Q_a \\ Q_b \end{pmatrix} = uU\begin{pmatrix} Q_a \\ Q_b \end{pmatrix}U^{-1} \tag{25}$$

where u is a two-dimensional matrix given by

$$u = \begin{pmatrix} A\lambda_1 & B\lambda_2 \\ B\lambda_1 & -A\lambda_2 \end{pmatrix} \tag{26}$$

Similar to Fan *et al.*, (1987) we introduce the following integral operator:

$$U = \left|\det u\right|^{1/2} \int\int_{-\infty}^{\infty} dq_a\, dq_b \left|u\begin{pmatrix} q_a \\ q_b \end{pmatrix}\right\rangle\left\langle\begin{pmatrix} q_a \\ q_b \end{pmatrix}\right| \tag{27}$$

where

$$\left|\begin{pmatrix} q_a \\ q_b \end{pmatrix}\right\rangle \equiv \left|q_a\right\rangle \otimes \left|q_b\right\rangle \tag{28}$$

is the two-mode coordinate eigenstate and the operators Q_a, Q_b satisfy the eigenstate equations

$$\begin{pmatrix} Q_a \\ Q_b \end{pmatrix}\left|\begin{pmatrix} q_a \\ q_b \end{pmatrix}\right\rangle = \begin{pmatrix} q_a \\ q_b \end{pmatrix}\left|\begin{matrix} q_a \\ q_b \end{matrix}\right\rangle \tag{29}$$

It is easy to check that

$$UU^\dagger = \left|\det u\right| \int\int_{-\infty}^{\infty} dq_a\, dq_b \int\int_{-\infty}^{\infty} dq_a'\, dq_b'$$

$$\times \left|u\begin{pmatrix} q_a \\ q_b \end{pmatrix}\right\rangle\left\langle\begin{pmatrix} q_a \\ q_b \end{pmatrix}\right|\left|\begin{pmatrix} q_a' \\ q_b' \end{pmatrix}\right\rangle\left\langle u\begin{pmatrix} q_a' \\ q_b' \end{pmatrix}\right|$$

$$= U^{\dagger}U = 1 \tag{30}$$

Thus, one see that U is unitary. Furthermore, since

$$\left\langle u\begin{pmatrix} q_a \\ q_b \end{pmatrix}\middle\|\begin{pmatrix} Q_a \\ Q_b \end{pmatrix}\right) = \left\langle u\begin{pmatrix} q_a \\ q_b \end{pmatrix}\middle\| u\begin{pmatrix} q_a \\ q_b \end{pmatrix}\right) \tag{31}$$

we have

$$\begin{pmatrix} Q_a \\ Q_b \end{pmatrix} U^{-1} = \left|\det u\right|^{1/2} \int\int dq_a\, dq_b \left\|\begin{pmatrix} q_a \\ q_b \end{pmatrix}\right)\left\langle u\begin{pmatrix} q_a \\ q_b \end{pmatrix}\middle\|\begin{pmatrix} q_a \\ q_b \end{pmatrix}\right)$$
$$= U^{-1}u^{-1}\begin{pmatrix} Q_a \\ Q_b \end{pmatrix} \tag{32}$$

As a result, we obtain

$$U\begin{pmatrix} Q_a \\ Q_b \end{pmatrix} U^{-1} = u^{-1}\begin{pmatrix} Q_a \\ Q_b \end{pmatrix} \tag{33}$$

and its inverse transformation

$$U^{-1}\begin{pmatrix} Q_a \\ Q_b \end{pmatrix} U = u\begin{pmatrix} Q_a \\ Q_b \end{pmatrix} \tag{34}$$

Thus U is indeed the operator leading to the transformation (25). From (21), (22), together with $\left|00\right\rangle\left\langle 00\right| = :\exp(-a^{\dagger}a - b^{\dagger}b):$ (Fan *et al.*, 1987), we can express (27) as

$$U = \lambda_1\lambda_2 \frac{\sqrt{\omega_a\omega_b}}{\pi} \int\int_{-\infty}^{\infty} dq_a\, dq_b :W: \tag{35}$$

where :: stands for normal product and the operator W is

$$W = \exp\left[-\frac{\omega_a}{2}(A\lambda_1 q_a + B\lambda_2 q_b)^2 + \sqrt{2\omega_a}(A\lambda_1 q_a + B\lambda_2 q_b)a^{\dagger} - \frac{a^{\dagger 2}}{2}\right.$$
$$- \frac{\omega_b}{2}(B\lambda_1 q_a - A\lambda_2 q_b)^2 + \sqrt{2\omega_b}(B\lambda_1 q_a - A\lambda_2 q_b)b^{\dagger} - \frac{b^{\dagger 2}}{2}$$
$$\left. - a^{\dagger}a - b^{\dagger}b - \frac{\omega_a}{2}q_a^2 + \sqrt{2\omega_a}q_a a - \frac{a^2}{2} - \frac{\omega_b}{2}q_b^2 + \sqrt{2\omega_b}q_b b - \frac{b^2}{2}\right] \tag{36}$$

With the help of the integral results

$$\int\int_{-\infty}^{\infty} dq_a\, dq_b \exp(\eta_1 q_a^2 + \eta_2 q_b^2 + \eta_3 q_a q_b + \eta_4 q_a + \eta_5 q_b)$$
$$= \frac{2\pi}{\sqrt{4\eta_1\eta_2 - \eta_3^2}} \exp\left(\frac{\eta_3\eta_4\eta_5 - \eta_1\eta_5^2 - \eta_2\eta_4^3}{\sqrt{4\eta_1\eta_2 - \eta_3^2}}\right) \tag{37}$$

after doing some lengthy but straightforward algebra, we obtain a complicated normal product form of the unitary operator:

$$U = 2\lambda_1\lambda_2\sqrt{\frac{\omega_a\omega_b}{L}} \exp\left[\frac{1}{L}(\gamma_1 a^\dagger a^\dagger + \gamma_2 b^\dagger b^\dagger + \gamma_3 a^\dagger b^\dagger)\right]$$
$$\times :\exp\left[\frac{1}{L}(\gamma_4 a^\dagger b + \gamma_5 b^\dagger a + \gamma_6 a^\dagger a + \gamma_7 b^\dagger b)\right]:$$
$$\exp\left[\frac{1}{L}(\gamma_8 aa + \gamma_9 bb + \gamma_{10}\, ab)\right] \tag{38}$$

where

$$L = \omega_a\omega_b(\lambda_1^2\lambda_2^2 + A^2\lambda_1^2 + A^2\lambda_2^2 + 1) + \omega_a^2 B^2\lambda_2^2 + \omega_b^2 B^2\lambda_1^2 \tag{39}$$

$$\gamma_1 = -\frac{L}{2} + \omega_a\omega_b(\lambda_1^2\lambda_2^2 + A^2\lambda_1^2) + \omega_a^2 B^2\lambda_2^2 \tag{40}$$

$$\gamma_2 = -\frac{L}{2} + \omega_a\omega_b(\lambda_1^2\lambda_2^2 + A^2\lambda_2^2) + \omega_b^2 B^2\lambda_1^2 \tag{41}$$

$$\gamma_3 = 2AB\sqrt{\omega_a\omega_b}(\omega_b\lambda_1^2 - \omega_a\lambda_2^2) \tag{42}$$

$$\gamma_4 = 2\lambda_2 B\sqrt{\omega_a\omega_b}(\omega_a + \omega_b\lambda_1^2) \tag{43}$$

$$\gamma_5 = 2\lambda_1 B\sqrt{\omega_a\omega_b}(\omega_b + \omega_a\lambda_2^2) \tag{44}$$

$$\gamma_6 = 2\lambda_1 A\omega_a\omega_b(\lambda_2^2 + 1) - L \tag{45}$$

$$\gamma_7 = -2\lambda_2 A\omega_a\omega_b(\lambda_1^2 + 1) - L \tag{46}$$

$$\gamma_8 = -\frac{L}{2} + \omega_a\omega_b(A^2\lambda_2^2 + 1) + \omega_a^2 B^2\lambda_2^2 \tag{47}$$

$$\gamma_9 = -\frac{L}{2} + \omega_a\omega_b(A^2\lambda_1^2 + 1) + \omega_b^2 B^2\lambda_1^2 \tag{48}$$

$$\gamma_{10} = 2\lambda_1\lambda_2 AB\sqrt{\omega_a\omega_b}(\omega_a - \omega_b) \tag{49}$$

Operating with U on $|00\rangle_{ab}$, we find a transformation from the phonon and photon vacuum state to the polariton vacuum state, i.e.,

$$\left|00\right\rangle_{\alpha\beta} = 2\lambda_1\lambda_2\sqrt{\frac{\omega_a\omega_b}{L}}\exp\left[\frac{1}{L}(\gamma_1 a^\dagger a^\dagger + \gamma_2 b^\dagger b^\dagger + \gamma_3 a^\dagger b^\dagger)\right]\left|00\right\rangle_{ab} \quad (50)$$

Combining (8) and (9), noting

$$A^2 + B^2 = 1, \qquad \frac{A^2}{f^2\lambda_2^4} + \frac{f^2B^2}{\lambda_1^4} = \frac{1}{f^2} \quad (51)$$

we can rewrite (39) as follows:

$$L = \omega_a\omega_b\lambda_1^2\lambda_2^2\left(\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2 f^2} + 1 - \frac{1}{f^2}\right)\left(1 + \frac{A^2}{\lambda_2^2} + \frac{f^2B^2}{\lambda_1^2}\right) \quad (52)$$

Thus we can simplify γ_1 in the form

$$\gamma_1 = -\frac{E_0 L}{2(E_0 + \omega_a)} \quad (53)$$

Similarly, we obtain

$$\gamma_2 = -\frac{E_0 L}{2(E_0 + \omega_b)}, \qquad \gamma_3 = \frac{E_0}{k}L \quad (54)$$

Recently, an interesting wave vector regarded as a new ground state of the polariton system was constructed (Wang *et al.*, 1997). We find that such a state is the same as the state $\left|00\right\rangle_{\alpha\beta}$ except for a normalized factor. Thus, in our view, the new ground state is just the polariton vacuum state. In what follows, we demonstrate that the polariton vacuum state is a squeezed state for the phonon and photon subsystem. To see this, using the transformations

$$\exp(-i\theta J_y)\, a^\dagger \exp(i\theta J_y) = a^\dagger \cos\frac{\theta}{2} + b^\dagger \sin\frac{\theta}{2} \quad (55)$$

$$\exp(-i\theta J_y)\, b^\dagger \exp(i\theta J_y) = b^\dagger \cos\frac{\theta}{2} - a^\dagger \sin\frac{\theta}{2} \quad (56)$$

we can express $\left|00\right\rangle_{\alpha\beta}$ as

$$\left|00\right\rangle_{\alpha\beta} = 2\lambda_1\lambda_2\sqrt{\frac{\omega_a\omega_b}{L}}\exp(i\theta J_y)\left|00\right\rangle_{\alpha'\beta'} \quad (57)$$

where $J_y = (ab^\dagger - a^\dagger b)i/2$ and $\exp(i\theta\, J_y)$ is a rotation operator. The state

$$\left|00\right\rangle_{\alpha'\beta'} = \exp(\sigma_1 a^\dagger a^\dagger + \sigma_2 b^\dagger b^\dagger)\left|00\right\rangle_{ab} \quad (58)$$

is a direct product of the two single-mode squeezed vacuum states whose parameters are given by

$$\tan\theta = \frac{\gamma_3}{\gamma_2 - \gamma_1} \tag{59}$$

$$\sigma_1 = \frac{1}{2L}[\gamma_1(1 + \cos\theta) + \gamma_2(1 - \cos\theta) - \gamma_3 \sin\theta] \tag{60}$$

$$\sigma_2 = \frac{1}{2L}[\gamma_1(1 - \cos\theta) + \gamma_2(1 + \cos\theta) - \gamma_3 \sin\theta] \tag{61}$$

To further show the properties of the rotated squeezed state, we now consider the zero-point fluctuation of the coupled system and focus on how to squeeze quantum noise in the two subsystems. Though the phonon and photon are not separable in the "dressed" phonon, we can still study the quantum noise in the quadrature variables of the two subsystems. We know that an apparent difference between quantum mechanics and classical mechanics is the presence of zero-point energy in a quantum mechanical system. This, of course, is consistent with the uncertainty principle according to which the internal coordinates of the system cannot all have their classical equilibrium values when the system is in the ground state. Thus, zero-point energy is often associated with the vacuum fluctuation of the field. To study the vacuum fluctuation of the polariton system, one needs to analyze the quadrature variables of the two subsystems. The quadratures are referred to the dimensionless coordinate and momentum, and are denoted by

$$X_a = \frac{a + a^\dagger}{\sqrt{2}}, \qquad Y_a = \frac{a - a^\dagger}{\sqrt{2}i} \tag{62}$$

$$X_b = \frac{b + b^\dagger}{\sqrt{2}}, \qquad Y_b = \frac{b - b^\dagger}{\sqrt{2}i} \tag{63}$$

It is easy to see that the quadrature operators satisfy the commutation relation $[X_j, Y_j] = i$ ($j = a, b$), which implies the uncertainty relation $\langle(\Delta X_j)^2\rangle\langle(\Delta Y_j)^2\rangle \geq 1/4$. As usual, the variance of the operator X is defined by the form $\langle(\Delta X)^2\rangle = \langle X^2\rangle - \langle X\rangle^2$. Due to the quantum coupling between the two subsystems, the measurement of some attribute of the phonon (or photon) provides information about the photon (or phonon). For a given state, the phonon subsystem exhibits squeezing if $\langle(\Delta X_a)^2\rangle < 1/2$ (or $\langle(\Delta Y_a)^2\rangle < 1/2$) and the photon subsystem does if $\langle(\Delta X_b)^2\rangle < 1/2$ (or $\langle(\Delta Y_b)^2\rangle < 1/2$). For the polariton vacuum state, we obtain

$$\langle(\Delta X_a)^2\rangle = \frac{1}{2}[(A_1 + A_2)^2 + (B_1 + B_2)^2] \tag{64}$$

$$\langle(\Delta Y_a)^2\rangle = \frac{1}{2}[(A_1 - A_2)^2 + (B_1 - B_2)^2] \tag{65}$$

$$\langle(\Delta X_b)^2\rangle = \frac{1}{2}[(A_3 + A_4)^2 + (B_3 + B_4)^2] \tag{66}$$

$$\langle(\Delta Y_b)^2\rangle = \frac{1}{2}[(A_3 - A_4)^2 + (B_3 - B_4)^2] \tag{67}$$

For simplicity, we consider the resonant case, i.e., $\omega_a = \omega_b = 1$. The transformation coefficients are then given by

$$A_1 = \frac{1}{2\sqrt{2}}[(1 + 2k)^{1/4} + (1 + 2k)^{-1/4}],$$

$$A_2 = \frac{1}{2\sqrt{2}}[(1 + 2k)^{1/4} - (1 + 2k)^{-1/4}] \tag{68}$$

$$B_1 = \frac{1}{2\sqrt{2}}[(1 - 2k)^{1/4} + (1 - 2k)^{-1/4}],$$

$$B_2 = \frac{1}{2\sqrt{2}}[(1 - 2k)^{1/4} - (1 - 2k)^{-1/4}] \tag{69}$$

$$A_3 = -B_1, \qquad A_4 = -B_2, \qquad B_3 = A_1, \qquad B_4 = A_2 \tag{70}$$

From (50), we obtain the transformation from the phonon and photon vacuum state to the polariton vacuum state

$$|00\rangle_{\alpha\beta} = N^{1/2}\exp\left(\frac{\mu}{2}a^{\dagger 2} + \frac{\mu}{2}b^{\dagger 2} + \gamma a^\dagger b^\dagger\right)|00\rangle_{ab} \tag{71}$$

where

$$N = \frac{2\sqrt{1 - 4k^2}}{(1 + \sqrt{1 - 2k})(1 + \sqrt{1 + 2k})} \tag{72}$$

$$\mu = -\frac{E_0}{E_0 + 1}, \qquad \gamma = -\frac{E_0}{k}, \qquad E_0 = -1 + \frac{1}{2}\sqrt{2 + 2(1 - 4k^2)^{1/2}} \tag{73}$$

For such a state, the corresponding fluctuations are given by

$$\langle(\Delta X_a)^2\rangle = \frac{1}{4}(\sqrt{1 + 2k} + \sqrt{1 - 2k}) \tag{74}$$

$$\langle(\Delta Y_a)^2\rangle = \frac{1}{4}\left(\frac{1}{\sqrt{1 + 2k}} + \frac{1}{\sqrt{1 - 2k}}\right) \tag{75}$$

$$\langle(\Delta X_b)^2\rangle = \frac{1}{4}(\sqrt{1+2k} + \sqrt{1-2k}) \tag{76}$$

$$\langle(\Delta Y_b)^2\rangle = \frac{1}{4}\left(\frac{1}{\sqrt{1+2k}} + \frac{1}{\sqrt{1-2k}}\right) \tag{77}$$

It can be seen from the above results that the quadrature variances $\langle(\Delta X_a)^2\rangle$ and $\langle(\Delta X_b)^2\rangle$ are reduced below the value 1/2, which means that squeezing always exists in the X_a and X_b directions for both the phonon and the photon subsystems, which shows that the polariton ground state includes the single-mode squeezed vacuum states of the two subsystems. The squeezing sensitively depends on the coupling strength k. The bigger the coupling strength, the better the squeezing.

In conclusion, we have investigated the Ghoshal–Chatterjee (1996) transformation and shown that the transformation can be expressed as an unitary operator form. A connection between the coupled and uncoupled states of the phonon–photon system is thus found. We also considered the quantum noise-induced effect when the polariton system is in its lowest energy state. Our results show that due to the interaction between the phonon and photon field, the subsystems of both the phonon and photon field may exhibit squeezing effects, while at the same time no radiation occurs because the coupled system is in its ground state. We have a situation where the whole system is in its lowest energy state while its two individual subsystems may possess nonclassical behavior. In addition, the transformation also shows clearly that a new ground state of the polariton system proposed by Wang *et al.* (1997) is just the polariton vacuum state.

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